MATH 5485 Introduction to Numerical Methods I

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Quiz 3 (take-home)

due October 4, 2013

Remember to:

- Work on your own.
- Justify your answers (especially when the answer is "yes" or "no", or a single number).
- Provide details (e.g., how to derive a solution).
- Do NOT use red color for your answers.
- Write legibly, especially the answers (if hand-written).

Option 2

Problem 1 (20pt). Sec. 2.3, Exercise 6.

Sol.

(a) p is a fixed point $\Leftrightarrow$ $p = 1 + p - \frac{1}{8}p^3$ $\Leftrightarrow$ $\frac{1}{8}p^3 = 1$ $\Leftrightarrow$ $p^3 = 8$ $\Leftrightarrow$ $p = 2$. Hence $p = 2$ is the only fixed point

(b) No, because $|g'(x)| = \left|1 - \frac{3}{8}x^2\right|$ cannot be bounded by $k < 1$. (That is, there is no such $k < 1$ that $|g'(x)| \le k$ for all x.)

(c) $|g'(2)| = 1 - \frac{3}{8}2^2 = -\frac{1}{2} \neq 0$, hence first order of convergence (see the theorem before the last theorem).

Problem 2 (10pt). Sec. 2.3, Exercise 10.

Sol.

$g(1/a) = (1/a)(2 - a(1/a)) = 1/a$, hence $1/a$ is a fixed point.

We have that $g'(x) = (2x - ax^2)' = 2 - 2ax$. Hence $g'(1/a) = 2 - 2a(1/a) = 0$. $g''(1/a) = -2a$. Hence the order of conv. is $\alpha = 2$ and asymp. err. const. is $\lambda = g''(1/a)/2 = -a$ (see the last theorem).

Problem 3 (20pt). Sec. 2.4, Exercise 4.

Sol.

(a) $(p_0, \dots, p_5) \approx (1., 1.28571, 1.19692, 1.17169, 1.16994, 1.16993)$.

(b) $\big((p_n - p)/(p_{n-1} - p)^2\big)_{n=1,\dots,5} \approx (4.00961, 2.01301, 2.41435, 2.55401, 2.56421)$ (compute this directly in Matlab, otherwise round-off errors will be huge). This clearly approaches $f''(p)/(2f'(p)) \approx 2.56425$

Problem 4 (10pt). Sec. 2.4, Exercise 8.

Sol.

$g(x) = x - f(x)/f'(x) = x - (1/x - a)/(-1/x^2) = x + (x - ax^2) = x(2 - ax)$.

Problem 5 (20pt). Sec. 2.5, Exercise 6.

Problem 6 (20pt). Sec. 2.6, Exercise 4.