MATH 5485 Introduction to Numerical Methods I

Instructor: *Alexander Shapeev* Fall 2013

Final Study Guide

1. Basic ideas:
   - order of convergence
   - asymptotic error constant
2. Rootfinding:
   (a) Basic ideas:
   - multiplicity of roots

   (b) Bisection method, False position, Newton's method, Secant method:
   - formulation of the algorithm
   - be able to compute a few iterations
   - requirements for convergence
   - order of convergence

   (c) Fixed point method in general:
   - requirements for existence of a fixed point
   - requirements for convergence
   - order of convergence
   - be able to compute a few iterations

   (d) Acceleration of convergence (Aitken's method):
   - application of an Aitken's method
3. Systems of equations:
   (a) Gaussian elimination
   - row operations
   - no pivoting, partial pivoting, scaled partial pivoting
   - formulation of the algorithm
   - application of the algorithm to a 2×2 or 3×3 matrix.

   (b) LU decomposition
   - via Gaussian elimination and via direct factorization
   - using LU factorization for solving systems
   - special matrices (diagonally dominant, positive definite, tridiagonal)
   - Cholesky decomposition
   - be able to find an LU decomposition of 2×2 or 3×3 matrix.

   (c) Iterative methods

- iteration matrix and requirements for convergence
- be able to write the iteration matrix and compute a few iterations for the Jacobi method, Gauss-Seidel method, and SOR.

(d) Newton's method

- formulation the method
- be able to compute a few iterations

4. Eigenvalues and Eigenvectors:

(a) Gershgorin's theorem

- be able to localize eigenvalues of a given matrix

(b) Power method, Inverse power method

- what method to use for the eigenvalue which has largest modulus/has smallest modulus/is closest to a given number
- general matrices vs symmetric matrices
- formulation of the method
- be able to compute a few iterations

(c) Deflation (Hotelling, i.e., symmetric only)

- Given A, λ_1, and v_1, determine the deflated matrix B.

5. Interpolation

(a) Lagrange interpolation

- Lagrange polynomials, Lagrange form of the interpolating polynomial

(b) Newton interpolation

- divided difference table, Newton form of the interpolating polynomial

(c) Optimal points of interpolation (only L_∞ norm), Chebyshev polynomials T_n

- definition, roots and other basic properties of T_n
- error estimates for optimal points

(d) Hermite interpolation

- be able to express H_i and $\hat{H}_i$ through $L_{n,i}$
- be able to write out the interpolating polynomial (through both the Lagrange or Newton approach).

(e) Piecewise linear and Hermite cubic interpolation

- given data, be able to write out interpolation function and evaluate it (or its derivatives) at certain points.

(f) Cubic spline interpolation

- be able to write out the system for c_j, solve small systems, and reconstruct $a_j/b_j/d_j$/function values on a certain interval or a certain point.

(g) Error estimates

- for each interpolation scheme, be able to evaluate or estimate the error of interpolation.