MATH 5485 Introduction to Numerical Methods I

Instructor: *Alexander Shapeev* September 27, 2013

Quiz 3 (take-home)

due October 4, 2013

Remember to:

- Work on your own.
- Justify your answers (especially when the answer is "yes" or "no", or a single number).
- Provide details (e.g., how to derive a solution).
- Do NOT use red color for your answers.
- Write legibly, especially the answers (if hand-written).

You can choose to complete either Option 1 or Option 2 or both. In the latter case your total points for the quiz will be calculated as a maximum over the two options.

Option 1

Problem 0 (100pt). The Lambert W function, also known as the product logarithm function, plays an important role in physics and other disciplines (see the respective Wikipedia entry). However, this function is not included in low-level programming languages like C or Fortran. Therefore, one may need to implement this function manually.

For non-negative x, Lambert W function, $w = w(x)$, can be defined as the solution to

$$w = \ln(x) - \ln(w),$$

where ln is the natural logarithm.

Your tasks are hence to

Task 1. Consider applying a simple iteration method with $g(w) = \ln(x) - \ln(w)$.

- Task 1.1. Find, theoretically, the range of values w (and hence, the range of x) for which the simple iterations converge and determine the order of convergence.
- Task 1.2. Write a Matlab code that uses the simple iteration method that attempts to evaluate this function to a relative accuracy of at least 10^{-12} and uses at most 100 iterations.
- Task 1.3. Test your code for values of x between 0.1 and 1000. Report for what range of values of x you observe convergence and the number of iterations needed to reach the accuracy of 10^{-12}. Comment on how these results compare to the answer from Task 1.1.

Task 2. (Newton's method).

Task 2.1. Write a Matlab code that uses the Newton's method to evaluate this function to a relative accuracy of at least 10^{-12} and uses at most 7 iterations.

Task 2.2. Use the four values of $x \in \{0.1, 1, 10, 100\}$ and report the error produced by your code, by comparing your results with the function `lambertw` provided in Matlab. (You are allowed to use Matlab's `lambertw` only for Task 2.2.)

Hint: You can use the following two facts to construct your initial guess:
(a) $w(x) \approx x$ for small (and positive) x, and
(b) $w(x) \approx \ln(x)$ for large x.

Option 2

Problem 1 (20pt). Sec. 2.3, Exercise 6.
Problem 2 (10pt). Sec. 2.3, Exercise 10.
Problem 3 (20pt). Sec. 2.4, Exercise 4.
Problem 4 (10pt). Sec. 2.4, Exercise 8.
Problem 5 (20pt). Sec. 2.5, Exercise 6.
Problem 6 (20pt). Sec. 2.6, Exercise 4.