Instructor: *Alexander Shapeev* October 26, 2013

Quiz 6 (take-home)

due November 1, 2013

First Name(s):

Last Name:

Remember to:

- Print this document out and give your answers here, except if you need to attach extra pages for lengthy details
- Work on your own.
- Justify your answers (especially when the answer is "yes" or "no", or a single number).
- Provide details (e.g., how to derive a solution).
- Do NOT use red color for your answers.
- Write legibly, especially the answers (if hand-written).

Weird but important: In all 4 problems each student will work with a different matrix A (one matrix per student), given by the following formula,

$$A = \begin{pmatrix} 1+u & -u & 0 & 0 \\ -u & u+v & -v & 0 \\ 0 & -v & v+w & -w \\ 0 & 0 & -w & w \end{pmatrix},$$

where u, v, w are given in your "secret link" (look for "Quiz 6"). To make sure you are using the right matrix, you can compute its smallest eigenvalue and compare it with the one given under your secret link. (Type "`min(eig(A))`" in Matlab for the smallest eigenvalue of `A`.)

Problem 1 (20pt). Consider the matrix A as described above.

(a) Is A strictly diagonally dominant? (5pt)

(b) Is A positive definite? (10pt)

(c) Is A tridiagonal? (5pt)

(Remember to give details for this and the following problems.)

Problem 2 (25pt).

(a) Compute the Cholesky decomposition of A (you can choose to give the exact, or a numerical answer).

(b) Using the Cholesky decomposition, solve the linear system $Ax = b$ with

$$b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Problem 3 (30pt). Solve the linear system $Ax = b$, same as in Problem 2(b), using

(a) Jacobi method

(b) Gauss-Seidel method

For each method: (1) Write down the iteration matrix T. (2) Take $x^{(0)} = 0$ and terminate iterations when $\|x^{(k+1)} - x^{(k)}\|_\infty$ falls below 10^{-4}. (3) Report the solution x and the number of iterations $k+1$.

(c) Give the details (here or on a separate paper) of how you obtained the answer (codes, Matlab commands, etc.). If you used the code from the book's author, you do not need to provide it here.

Problem 4 (25pt).

(a) Determine the optimal parameter ω for applying the SOR method to the matrix A.

(b) Solve the system $Ax = b$, same as in Problem 2(b), using the SOR method. As earlier, take $x^{(0)} = 0$ and terminate iterations when $\|x^{(k+1)} - x^{(k)}\|_\infty$ falls below 10^{-4}. Report the solution x and the number of iterations $k + 1$.

(c) Give the details (here or on a separate paper) of how you obtained the answer (formulae, codes, Matlab commands, etc.). If you used the code from the book's author, you do not need to provide it here.